Assignment 2.

(1) $f(x) = \begin{cases} \frac{x e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$\Rightarrow$ $\lim_{h \to 0^+} LHL = \frac{(-h) e^{1/(-h)}}{1+e^{1/(-h)}}$

$= \frac{(-h)}{e^{1/h}+1} = 0$]

RHL $= \frac{(h) e^{1/(h)}}{1+e^{1/(h)}} = 0$ LHL = RHL = f(0) = 0

$\therefore$ limit exist & function is continous.

(2) $f(x) = \begin{cases} \frac{\sin 2x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$\lim_{h \to 0^+} LHL = \frac{\sin 2(-h)}{(-h)} = \frac{\sin 2h}{h}$

$= 2\left(\frac{\sin 2h}{2h}\right) = 2$]

$\lim_{h \to 0^+} RHL = \frac{2 \sin 2h}{2(h)} = 2$] LHL = RHL $\neq$ f(0) = 1

limit exist but the function is discontinous at $x=0$ & it is removable discontinuity. We can make cont. by $f(0)=2$]

(3) $f(x) = \begin{cases} -x^2 & x \leq 0 \\ 5x-4 & 0 < x \leq 1 \\ 4x^3-3x & 1 < x < 2 \\ 3x+4 & x \geq 2 \end{cases}$

At $x=0$

LHL $= -(0)^2 = 0$

RHL $= 5(0)-4 = -4$

$f(0) = 0$

Discontinous at $x=0$ from RHL]

At $x=1$

LHL $= 5(1)-4 = 1$

RHL $= 4(1)-3(1) = 1$

$f(1) = 1 \Rightarrow$ cont. at $x=1$]

At $x=2$

LHL $= 4(2)^3 - 3(2) = 26$

RHL $= 3(2)+4 = 10$

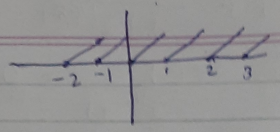
$f(2) = 10$ Discontinous at $x=2$ from LHL]

(4) $f(x) = \frac{x-|x|}{x}$

$\lim_{h \to 0^+} LHL = \frac{(-h)-|-h|}{-h} = \frac{-h-h}{-h} = 2$ $\lim_{h \to 0^+} RHL = \frac{(0+h)-|h|}{h} = 0$

$f(0) = 0$ LHL $\neq$ RHL Discontinous at $x=0$ from LHL]

(5). $f(x) = x - [x]$ at $x = 3$.



$\lim_{h \to 0^+} LHL = -h - [-h] = -(-1) = 1$; $\lim_{h \to 0^+} RHL = h - [h] = 0$

LHL ≠ RHL discontinous at $x = 3$]

(6)

$$f(x) = \begin{cases} (1+x)^{\frac{1}{x}} & x \neq 0 \\ e & x = 0 \end{cases}$$

$\lim_{x \to 1} f(x) = e^{(1+x)-1)\frac{1}{x}}$

$= e$]

$\lim_{h \to 0^+} LHL = (1-h)^{\frac{1}{-h}} = \frac{1}{(1-h)^{\frac{1}{h}}} = \frac{1}{e} \neq 0$]

$\lim_{h \to 0^+} RHL = (1+h)^{\frac{1}{h}} = e$]

$e = LHL = RHL \neq f(0) = 1$

Discontinous at $x = 0$]

(7)

(i) $$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\lim_{h \to 0^+} LHL = \sin\left(\frac{1}{-h}\right) = -\sin\left(\frac{1}{h}\right)$]

$RHL = \sin\left(\frac{1}{h}\right)$]

LHL ≠ RHL ≠ f(0) = 0 Discontinous at $x = 0$]

(ii) $$f(x) \begin{cases} x\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

$\lim_{h \to 0^+} LHL = (-h)\sin\left(\frac{1}{-h}\right) = 0$]

$\lim_{h \to 0^+} RHL = (h)\sin\left(\frac{1}{h}\right) = 0$]

LHL = RHL = f(0) = 0 continous at $x = 0$]

(8). $\sin x$, x

(9) ~~Dirichlet function~~

(10).

$$f(x) = \begin{cases} x & x \text{ is rational} \\ -x & x \text{ is irrational} \end{cases}$$